

Studenti plusdotati, approccio al compito e relazioni tra pari durante i compiti di matematica

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Sommario

Il presente caso di studio su due bambini di scuola primaria mira a esplorare l'approccio al compito durante l'esecuzione di compiti di matematica impegnativi, in particolare i problemi di logica e i cosiddetti problemi impossibili in un contesto extrascolastico. La metodologia di osservazione adottata è stata attiva e intenzionale, condotta dai tutor, utilizzando suggerimenti mirati a modificare l'approccio alla gestione del compito. Osservazioni video e note di campo sono state utilizzate per tracciare, interpretare e raccogliere informazioni sui comportamenti verbali e non verbali. Il lavoro evidenzia l'impatto della scolarizzazione sull'approccio al compito, portando all'adozione di schemi rigidi nell'esecuzione di compiti matematici nei bambini plusdotati. Si evidenziano inoltre le strategie e gli stili adottati nonché l'importanza dell'interazione tra pari all'interno di attività su interessi comuni. Si intende, infine, evidenziare il ruolo del mentore/tutor adulto nella mediazione e nella promozione del pensiero divergente, anche nelle attività di peer tutoring con bambini plusdotati.

Parole chiave

Plusdotazione matematica, Pensiero, Interazioni tra pari, Scolarizzazione, Scuola primaria.

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Gifted students, approach to task and peer relationships during math tasks¹

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Abstract

The present case study of a pair of primary school children explores their approach to tasks during the execution of challenging math tasks, particularly logic problems and the so-called impossible problems in an extracurricular context. The observation methodology was active and intentional and conducted by tutors using prompts aimed at generating change in the way the task was handled. Video observations and field notes were used to track, interpret, and collect information on verbal and nonverbal behaviours. The work highlights the impact of the schooling process on task approach, leading to the adoption of rigid patterns in the performance of mathematical tasks in gifted children. We also highlight the strategies and styles they adopt and the importance of peer interaction within activities on common interests. Finally, we want to emphasize the importance of the role of mediation and promotion of divergent thinking played by the adult mentor/tutor even in peer tutoring activities with gifted children.

Keywords

Math Gifted, Thinking, Peer interactions, Schooling, Primary.

¹ The article is the result of the authors' collective work; however, paragraphs *Highly Able Mathematics Students and Challenging Tasks; The case study: children, setting and materials; The didactic intervention; Results and Conclusion* can be attributed to Clarissa Sorrentino, paragraphs *Introduction and Results and Conclusion* to Francesca Baccassino, paragraphs *Description of the cases and Results and Conclusion* to Marina De Nunzio.

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Introduction

There are different definitions of mathematical giftedness related to the specificity of the environmental and temporal context, but also to the multiplicity of conceptions of mathematics itself. We can consider mathematical giftedness as «the ability for rapid and broad generalization of mathematical relationships and operations and flexibility of mental processes» (Singer et al., 2016, p. 3). Therefore, students can demonstrate their excellent competence in mathematics both through performance, i.e., the speed of acquiring new information and performing well on school and academic tests, and through the «ability to create new processes and products» (p. 4) and to seek creative solutions to solve problems, negotiating solutions and engaging divergent thinking beyond the school subject. «Being gifted in mathematics does not necessarily lead to high achievement in this subject, while high achievement in mathematics does not necessarily mean being mathematically gifted» (p. 4).

Among the cognitive characteristics of math-gifted children there is the ability to memorize mathematical questions by drawing on identified structures, the ability to construct and use mathematical structures, the ability to switch from one mode of representation to another, the ability to reverse lines of thought, ability to grasp complex structures and work with them, ability to construct and use mathematical analogies, mathematical sensitivity, mathematical creativity (Assmus, 2016), high ability to identify patterns and abstract thinking; flexibility and creativity in approaching problem solving; ability to transfer mathematical concepts to an unfamiliar situation; persistence and resilience in solving challenging problems (Stepanek, 1999).

Gifted students who primarily use flexible and creative solution strategies often underachieve (Reis & McCoach, 2000) in school and escape identification by running the risk of potential dispersion. This happens because school teaching focuses on logical and computational activities that involve rigid processes and one-size-fits-all solutions, neglecting activities focused on flexible, creative, and divergent thinking patterns. Often, in other words, mathematics instruction, especially in the early years of schooling, provides little space for creativity. «The “problems” proposed to pupils are nothing more than disguised exercises in which the pupil is required to identify the solution (unique and codifiable in one way, often translatable into a calculation)» (De Nunzio, 2019, p. 162). Gifted children may feel different from their peers and learn to mask their abilities (Gross, 2003) and often prefer to work individually, in pairs or small groups with students who have similar interests.

However, even in pairs or interest groups, learning cannot be left to chance but must be structured by a qualified mentor or tutor. In fact, a task that is too free and unstructured would bring to light difficulties found in the population of

gifted children such as problems of self-regulation, imprecision of the solutions provided and difficulties of expression due to speed of thought, presumption of knowledge and impulsivity, problems of action planning due to the branching of ideas that does not allow the determination of which data to prioritize (Siaud-Facchin, 2008), reluctance towards knowledge not deemed useful or interesting, intolerance towards errors and difficulty accepting what appears illogical.

Highly Able Mathematics Students and Challenging Tasks

The teaching habit that leads students to approach mathematical tasks through a linear data-operation-solution process discourage convergent thinking and does not produce significant learning. In opposite line with this practice, teachers can propose challenging tasks that do not involve a rigid resolution procedure and a single solution. Research suggest that highly able mathematics students demonstrate lower «threshold» of boredom, this explains why it's very important to continuously challenge them to gain and sustain their motivation (Barbier et al., 2022; Smedsrud, Nordahl-Hansen & Idsøe, 2022).

A possible solution to this is to design tasks that stimulate children at an optimal level, and it can be possible only after a profound knowledge of students, their math ability level, their cognitive and learning preferences (Smedsrud, Nordahl-Hansen & Idsøe, 2022; Westgate & Wilson, 2018). When children look for a solution to a nonstandard problem, their expertise and creativity interact making links to the unknown (Singer & Voica, 2016). «Problem solving, that is seeking a solution to a mathematical situation for which students have no immediately obvious process or method, is often cited as a major goal in any mathematics program» (Singer et al., 2016, p. 17). However, students should «not only learn to solve problems, but also to rephrase and pose new questions that are authentic problems for themselves, challenging them to persevere and struggle to find a solution» (Singer et al., 2016, p. 18). Providing challenging mathematical tasks helps students increase their mathematical talents while reducing the difficulties of self-regulation and impulsivity described above. This is because solutions to challenging tasks «often involve explanations, multiple strategies, models and tools, questioning, conjecturing, and ongoing evaluation» (p. 20).

Challenges might be problem posing and open-ended tasks and impossible problems that are unreal problems whose text describes a situation that does not adhere to reality, contradictory i.e., vitiated by formal contradictions concealed in the data or text, insoluble to which it is impossible to give a solution due to lack of data or other vices (D'Amore & Sandri, 1993, 1998).

This breaks the school conditioning that distances mathematics from reality due to the stereotypical nature of problems proposed by textbooks that impose

implicit rules and exclusions of realistic considerations (Verschaffel, Greer & De Corte, 2000; Schoenfeld, 1991).

The case study: children, setting, and materials

The study presented here was driven by the need to investigate how two mathematically gifted children approach the task. In particular, we wanted to answer the following questions: Q1 How are cognitive, social, and affective aspects connected in gifted students? Q2 What thinking patterns (e.g., speed, depth, style, type) are evident in highly able mathematics students during open learning tasks? Q3 What role do mentors play in supporting the development of mathematical talent? Q4 Can the process of formal schooling in mathematics be a limitation in the expression of mathematical thinking?

We addressed these questions through participant observation. According to the methodology, active and intentional observation by the tutors with the use of prompts aimed at generating a change in the way the task was used. To track, interpret and collect information on non-verbal behaviour, video observations and field notes were used.

Description of the cases

The children who participated in the study were D, 6 years and 11 months old, and G, 8 years old. Both are gifted math children. D had just finished the first year of primary school, while G had completed the third year of primary school. The children were considered gifted in mathematics on the basis of interviews with parents and teachers, and the analysis of school performance.

In particular, the Italian adaptation by Fabio (2019) of Sommer, Fink and Neubauer's (2008) Checklist for the identification of giftedness, which calculates indices inherent to the intellectual, social, creative and global giftedness areas, was administered to parents and teachers. In addition to the information provided by the teachers on school performance, the parents' statements in terms of their knowledge of numbers and quantities at the age before 3 years were also examined.

In the case of D, the *Checklist for the identification of giftedness* (Fabio, 2019) was administered to parents and showed the presence of creative and global giftedness.

The psychodiagnostics assessment was also considered the *WISC IV multi-componential test*, which reports a *Visual Spatial Index* (VSI) of 130 (percentile rank 97.9) higher than the average of the reference sample.

A pedagogical assessment of the child's mathematical giftedness was also conducted by the same educators who led the university lab. In three meetings lasting approximately one hour each, the child's abilities were examined with a focus on calculation skills. The child was tested on the *Potential Intelligence Test* (Fabio, 2007), an instrument for measuring cognitive modifiability. The child reported an IMG (*General Modifiability Index*) of 53, which indicates a highly modifiable learner. In particular, he was fast and correct in tasks concerning inductive reasoning ability and the completion of number series.

For the assessment of calculation skills, he was also assigned some tests of the *AC-MT 6-11 Test* (Cornoldi, Lucangeli & Bellina, 2012) in a scaled manner from the first grade of primary school upwards.

The tests revealed advanced skills in strategic thinking and mental calculation: the child correctly performs all the tasks in a shorter time than is considered optimal for children older than his age.

In the case of G., the *Checklist for the identification of giftedness* (Fabio, 2019) was administered to parents and teachers and both revealed the presence of creative and global giftedness. As far as the *Potential Intelligence Test* (Fabio, 2007) is concerned, the child reported an IMG (*General Modifiability Index*) of 58, which indicates a highly modifiable learner.

The didactic intervention

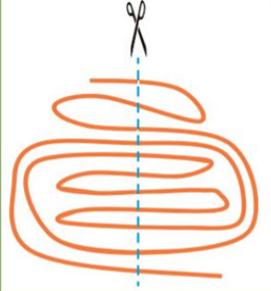
Two-hour meetings were held. The children were welcomed inside a university laboratory where there were ample spaces to move around freely. During the first meeting, the activities included a sequence of exercises already established by the tutors.

During the second meeting, in addition to the math activities proposed by the tutors, the children could choose to play with other games present in the room. The activities proposed to the children were taken from the Kangaroo math competitions (an international math competition that takes place every year in many countries around the world, including Italy). The problems proposed were a series of international mathematical challenges that tested the logical reasoning, problem-solving skills and creativity of the participants.

Furthermore, two impossible problems were proposed. The chosen Kangaroo activity started with so-called 3-point-problems, warm-up activities «meant to be one-step-problems [...] solvable for the vast majority of the pupils [and ended with 5-point-problems] meant for the mathematically more gifted students and should give them something to think about» (Akveld, Caceres-Duque & Geretschläger, 2020, p. 54). In figure 1 and 2 we show some of 3-point problems proposed, while in figure 3 we show a 5-point-problems.

Figure 1

9 Edward cut a ribbon as shown in the picture. In the end, how many pieces of tape did you get?



9 10 11 12 13

A B C D E

Example n. 1 of the 3-point-problems proposed.

Figure 2

6 The pink tower is taller than the blue tower but lower than the green tower. The silver tower is taller than the green tower. What is the tallest tower?



A The pink tower.

B The green tower.

C The blue tower.

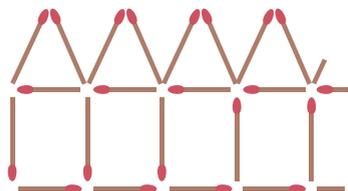
D The silver tower.

E It's impossible to decide.

Example n. 2 of the 3-point-problems proposed.

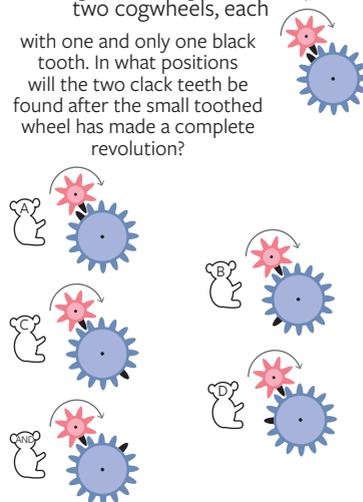
Figure 3

Sofia built a row of 10 houses using matches. In the figure you can see the beginning of the line. How many matches did Sofia use in total?



- A) 50
- B) 51
- C) 55
- D) 60
- E) 62

The image shows a gear made up of two cogwheels, each with one and only one black tooth. In what positions will the two black teeth be found after the small toothed wheel has made a complete revolution?



Example of the 5-point-problems proposed.

Results and Conclusion

The observation of the children in the study, through the videorecording, during and following the activities, allowed us to gain some insights into their cognitive and relational aspects. There are some common and different characteristics between the two children. Both demonstrate impulsivity in approaching the problems, often skipping or reading the command quickly, which led to wrong results, especially in the older child.

The speed in approaching the task was therefore a common characteristic of both children, but it proved counterproductive. Another common element was the preference for facing mathematical problems rather than playing with other toys available (castle with knights, crayons, etc.).

However, there are also some differences in the approach to the task: G (the older child) tended to underestimate the difficulty of the task, while D showed insecurity. G, as soon as a question was presented to him, defined it as «very easy» and wanted to explain to D how to reach the solution. D, on the other hand, while initially arriving at appropriate answer hypotheses or intervention strategies, later tended to embrace the solutions proposed by G. Another important difference concerns the strategies used to face the activities: G, who was more «schooled/educated», used more structured strategies but fell into the trap of the «age of the captain» task. This expression is used to indicate the behaviour of a student who calculates the answer to a problem using all or some of the

numbers present in the statement, even if this problem cannot lead to a numerical solution (Adda, 1987).

D, on the other hand, who had only one year of schooling, had not yet tackled some aspects of calculation (multiplications and divisions) and therefore used more naive theories that sometimes led to correct results. This emerged both in facing impossible problems and some Kangaroo activities. It is therefore interesting to note that even gifted children, once included in the school context, can be subject to the so called «didactical contract», following precise steps in the execution of a sequence of tasks (Brousseau, 2020).

In the past, Brousseau explained that:

In a teaching situation, prepared and implemented by a teacher, the student generally has the task of solving a (mathematical) problem presented to him, but access to this task is achieved through an interpretation of the questions asked, the information provided, the obligations imposed which are constants in the teacher's way of teaching. These (specific) habits of the teacher expected by the student and the behaviors of the student expected by the teacher constitute the teaching contract (Brousseau, 1980, p. 127).

And become so ingrained in children that they are also used in extracurricular contexts. It is therefore plausible to think that the school setting models the task approach, neutering that sense of creativity necessary for solving mathematical problems even in children who have the potential to find the solutions. The study, although conducted with only two children, also finally highlights the very important role of the mediator, who must build an adequate context and have a careful eye on the interpersonal dynamics that are triggered in peer activities, which require continuous metacognitive reflection on the task and recall of what is done, on-going and after the activities.

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